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$$v_x = \partial \varphi / \partial x, \quad v_y = \partial \varphi / \partial y, \quad v_z = \chi(t) + h(f, g) \quad (1)$$

and the vortex field will be:

$$\Omega_x = \frac{\partial h}{\partial f} \cdot \frac{\partial f}{\partial y} + \frac{\partial h}{\partial g} \cdot \frac{\partial g}{\partial y}, \quad \Omega_y = -\frac{\partial h}{\partial f} \cdot \frac{\partial f}{\partial x} - \frac{\partial h}{\partial g} \cdot \frac{\partial g}{\partial x}, \quad \Omega_z = 0. \quad (2)$$

Pressure is defined by the formula:

$$p = c(t) - \rho \frac{\partial \varphi}{\partial t} + \rho U - \rho \left(\frac{1}{2} \left| \frac{dF}{d\zeta} \right|^2 + z \frac{dx}{dt} \right) \quad (3)$$

and the surfaces on which the particles of the fluid move will be:

$$(1) \text{ wings: } f(x, y; t) = a, \quad g(x, y; t) = b \quad (4)$$

$$(2) \text{ body: } z = \int \chi(t) dt + t \cdot h(f, g) + q(f, g) \quad (5)$$

where χ, c, h, q are arbitrary functions.

Theorem 2

To an analytic function of a complex variable $F(\zeta) = \varphi + i\psi$ corresponds a class of three-dimensional stationary flow of an ideal incompressible homogeneous fluid:

$$v_x - i v_y = dF/d\zeta, \quad v_z = \theta(\psi) \quad (\theta: \text{arbitrary function}) \quad (6)$$

Flow 6 is everywhere vortical:

$$\Omega_x - i \Omega_y = \frac{dF}{d\zeta} \cdot \frac{d\theta}{d\psi}, \quad \Omega_z = 0. \quad (7)$$

Pressure:

$$p = c + \rho U - \frac{1}{2} |dF/d\zeta|^2 \quad (8)$$

the flow surfaces consist of two families:

$$(1) \text{ wing: } \psi(x, y) = a.$$

$$(2) \text{ body: } z = \theta(\psi) \int \left\{ |d\zeta| |dF|^2 dF \right\}_{\psi=a} + \omega(\psi). \quad (9)$$

Theorem 3

In order that the surface $z = V(x, y) + c$ can be considered as a fuselage for a given wing $\psi(x, y) = a$, it is necessary and sufficient that the function V be satisfied by the following equation:

$$D \{ D(V, \psi) / D(x, y), \psi \} / D(x, y) = 0. \quad (10)$$

Theorem 4

The set of all fuselages, represented parametrically:

$$x = \alpha + \partial W / \partial \beta, \quad y = \beta - \partial W / \partial \alpha, \quad z = \alpha - \partial W / \partial \beta \quad (11)$$

possesses the property that the function $W(\alpha, \beta)$ satisfies the following 4th-order partial differential equation:

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$$D(\Sigma, \sigma) / D(\alpha, \beta) = 0 \quad (12)$$

where $\sigma = \beta + \partial W / \partial \alpha$, $\Sigma = A/B$

$$A = \frac{2}{\Delta} \left[\left(1 - \frac{\partial^2 W}{\partial \alpha \partial \beta} \right) \frac{\partial}{\partial \alpha} \frac{1}{\Delta} \frac{\partial^2 W}{\partial \alpha^2} - \frac{\partial^2 W}{\partial \beta^2} \frac{\partial}{\partial \alpha} \frac{1}{\Delta} \left(1 + \frac{\partial^2 W}{\partial \alpha \partial \beta} \right) \right] +$$

$$\frac{\partial}{\partial \beta} \left[\left(\frac{1}{\Delta} \frac{\partial^2 W}{\partial \alpha^2} \right)^2 + \frac{1}{\Delta^2} \left(1 + \frac{\partial^2 W}{\partial \alpha \partial \beta} \right)^2 \right]$$

$$B = 1 + \frac{4}{\Delta^2} \left[\left(1 + \frac{\partial^2 W}{\partial \alpha \partial \beta} \right)^2 - \Delta \left(1 + \frac{\partial^2 W}{\partial \alpha \partial \beta} \right) + \left(\frac{\partial^2 W}{\partial \alpha^2} \right)^2 \right]$$

$$\left(\text{where } \Delta = 1 + \frac{\partial^2 W}{\partial \alpha^2} \cdot \frac{\partial^2 W}{\partial \beta^2} - \left(\frac{\partial^2 W}{\partial \alpha \partial \beta} \right)^2 \right).$$

Examples

Consider the function $F = \frac{m}{2\pi} \ln \zeta$, corresponding to a source in two-dimensional motion. Here the wing can be taken as the zx -plane, and the fuselage as a paraboloid of rotation. The flow lines are defined by the intersection of the planes, passing through the z -axis, with the paraboloid. The projection of flow on the xy -plane is the well-known potential two-dimensional flow near the source. [The original article, available in CIA, shows a photograph of a model of the flow surfaces for this function.]

Consider another simple example: $F = \frac{\Gamma}{2\pi i} \ln \zeta$. Here the wings are circular cylinders (with the z -axis), and the fuselage is a helicoid. The projection of flow on the xy -plane is the well-known planar flow near a point source: [The original gives a photograph of the model.]

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